Journal of Sound and Vibration 326 (2009) 177–189

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Identification of local damages in coupled beam systems from measured dynamic responses

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ARTICLE INFO

Article history:
Received 27 November 2008
Received in revised form
19 February 2009
Accepted 18 April 2009
Handling Editor: L.G. Tham
Available online 15 May 2009

ABSTRACT

This paper deals with local damages identification for strongly and weakly coupled beam systems with close and repeated natural frequencies based on the measured dynamic responses of the systems under external moving forces. The dynamic responses of the coupled beam systems are calculated from Newmark integration method and they are used for structural damage detection. The mode localization phenomenon due to local damage(s) in the weakly coupled beam system is studied. In the inverse analysis, a dynamic response sensitivity-based finite element model updating method is employed for the detection of local damage(s) in the coupled beam systems. Numerical simulation shows that the proposed method is effective in identifying the damages in the coupled beam systems with good accuracy from several measurements. It is found the proposed method is insensitive to artificial measurement noise and has the potential for practical applications.

1. Introduction

Structural damage detection using the dynamic characteristic parameters and dynamic responses has been a hot research topic in the past few decades. A direct, fast and inexpensive method is therefore required to evaluate and localize damage using the changes in dynamic responses between the intact and damaged states of the structure. There are a lot of non-destructive methods in the literature for structural damage detection. Doebling et al. [1] provided a comprehensive review on the damage detection methods by examining changes in the dynamic properties of a structure. Housner et al. [2] presented an extensive summary on the state-of-the-art methods in the control and health monitoring of civil engineering structures. Zou et al. [3] summarized the methods on vibration-based damage detection and health monitoring for composite structures, especially in delamination modeling techniques and delamination detection.

Damage detection usually requires a mathematical model on the structure in conjunction with experimental modal parameters of the structure. The identification approaches are mainly based on the change in the natural frequencies [4–6], mode shapes [7–9] or measured modal flexibility [10–13]. The natural frequency is easy to measure with a high level of accuracy, and is the most common dynamic parameter for damage detection. However, problems may arise in some structures if only natural frequency is used, since the symmetry of the structure would lead to non-uniqueness in the solution in the inverse analysis of damage detection. This problem can be overcome by incorporating the mode shape data in the analysis. Finite element model updating method is the most popular tool for damage detection making use of these modal parameters. A large number of gradient-based finite element model updating methods have been discussed by

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2009.04.028

Friswell and Mottershead [14], and they have been used in damage detection of structures [15–20]. A two-stage method has been proposed to differentiate the local damage and model error in the structure [13,21]. The finite element model of the undamaged structure is firstly updated to remove most of the model errors to have a more accurate model. Then the differences in the modal parameters between the damaged and the intact structures are used to estimate the changes in the system parameters.

There are also literatures on damage detection in time domain using structural dynamic response. Cattarius and Inman [22] used the time histories of vibration response of the structure to identify damage in smart structures. Majumder and Manohar [23,24] proposed a time domain approach for damage detection in beam structures using vibration data. The force induced by a vehicle moving on the bridge was taken to be the source of excitation. This paper also makes use of the dynamic responses from the moving forces. Practically speaking, it is difficult to excite a large civil structure such as a long span bridge using an exciter, so moving forces will be more suitable to be used as excitation force rather than the sinusoidal force from an exciter. More recently, Chen and Li [25] and Shi et al [26] presented methods to identify structural parameters and input time history from output-only measurements iteratively. Law and Zhu [27] proposed an approach for damage detection in a concrete bridge structure in time domain.

To the authors' knowledge, few papers in the literature deal with damage detection in the coupled beam systems [28,29]. However, they did not deal with the case when the systems with close and repeated natural frequencies. As we know, when the structures have repeated or close eigenvalues, in the damage detection, it may cause ill-conditioning or rank deficiency in the solution [30]. This paper aims to identify the local damages in the coupled beam systems from the response sensitivity-based finite element method using the measured dynamic responses in time domain. The systems with both close and repeated natural frequencies for local damage detection will be investigated. Local damage in the system is introduced as a reduction in the stiffness of individual elements (i.e. a reduction of the flexural rigidity EI), but the other properties remain unchanged. This is similar to Refs. [31,32] by introducing the concept of the damage parameter in each element. First of all, free vibration analysis is carried out for the coupled beam system from the finite element analysis. The mode localization phenomenon is investigated in the weakly coupled beam system due to the disorder caused by local damages. And the dynamic responses of the system are calculated from the direct integration method. Then, an inverse problem is conducted to identify the local damages using response sensitivity-based finite element model updating method in the strongly and weakly coupled systems and the effect of artificial measurement noise on damage detection is investigated.

2. Forward analysis

2.1. Finite element model of the coupled beam system

Fig. 1 shows a general coupled beam system that consists of two beams arbitrarily coupled via a set of linear and rotational springs. The use of the coupling springs can take into account the effects of many non-rigid connectors such as point welds and bolt joints which are necessary in practice. When the coefficients of the coupling spring k_s and k_r are small, the system is strongly coupled, and when the coefficients k_s and k_r are large, the system is weakly coupled. In addition, the beams are generally supported on a set of elastic restraints at the boundary ends. So, the traditional homogeneous boundary conditions can be directly obtained from this general boundary condition by accordingly setting the stiffness constants of the springs to zero or infinity. In the finite element model of the damaged systems, the local damage is modeled as a reduction in the elemental flexural rigidity EI, but the other properties remain unchanged.

The equation of motion of the system under N_f moving forces by general finite element representation can be written as

$$\mathbf{M}\{\ddot{d}\} + \mathbf{C}\{\dot{d}\} + \mathbf{K}\{d\} = \mathbf{T}\{F(t)\}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$ are the system mass matrix, stiffness matrix and damping matrix, respectively, Rayleigh damping model is adopted in the study, i.e., $\mathbf{C} = a_1\mathbf{M} + a_2\mathbf{K}$, where a_1 and a_2 are constants to be determined from two given damping ratios that corresponding to two unequal modal frequencies of vibration. $\mathbf{T}F(t)$ is the equivalent nodal force vector from the

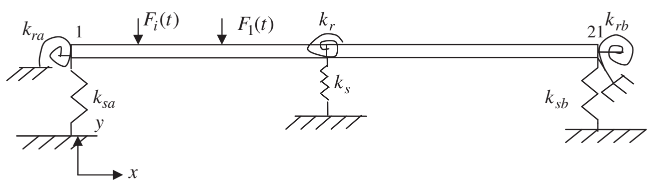

Fig. 1. The dual-span coupled beam system with elastic supports under moving loads (1, 21: node number of FEM).

moving force with

$$\mathbf{T} = \begin{bmatrix} 0 & \cdots & 0 & T_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & T_i & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & T_{Nf} & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

where $\mathbf{T}$ is a $N \times N_f$ matrix with zero entries except at the degrees-of-freedom corresponding to the nodal displacements of the beam elements on which the load is acting, and N is the number of degrees-of-freedom of the system after considering the boundary condition. The components of the vector T_i evaluated for the ith moving force on the jth element is given by the shape function

$$T_i = \left\{ \begin{array}{c} 1 - 3\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l}\right)^2 + 2\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l}\right)^3 \\ (\bar{x}_i(t) - (j-1)l)\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l} - 1\right)^2 \\ 3\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l}\right)^2 - 2\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l}\right)^3 \\ (\bar{x}_i(t) - (j-1)l)\left(\dfrac{\bar{x}_i(t) - (j-1)l}{l}\right)^2 - \dfrac{(\bar{x}_i(t) - (j-1)l)^2}{l} \end{array} \right\}, \tag{3}$$

with $(j-1)l \leqslant \bar{x}_i(t) \leqslant jl$, $\bar{x}_i(t)$ is the location of the ith moving force, l is the length of the finite element.

Neglecting the damping of system and setting the nodal force vector to be null, one will obtained the equation of motion of free vibration of the system and the natural frequencies and associated mode shapes of the system can be obtained from the eigenvalue analysis.

For a given set of moving forces, dynamic response of the system can be calculated from Eq. (1) using direct integration method.

3. Inverse problem

3.1. Dynamic response sensitivity with respect to the system parameters

In this paper, the local damages are simulated as reductions of the flexural rigidity EI in some elements of the system. In the inverse analysis, the local damages in the system are identified using dynamic response sensitivity-based finite element model updating method proposed by the authors [33]. The method is briefly reviewed as follows.

The dynamic response of the system is obtained from Eq. (1) from direct integration method, and then by taking partial derivative to a certain damage parameter, i.e., the elemental flexural rigidity EI, we have

$$\mathbf{M}\left\{\frac{\partial \ddot{d}}{\partial EI^i}\right\} + \mathbf{C}\left\{\frac{\partial \dot{d}}{\partial EI^i}\right\} + \mathbf{K}\left\{\frac{\partial d}{\partial EI^i}\right\} = -\frac{\partial \mathbf{K}}{\partial EI^i}\{d\} - a_2\frac{\partial \mathbf{K}}{\partial EI^i}\{\dot{d}\} \quad (i = 1, 2, \ldots, n), \tag{4}$$

where n is the number of the finite elements of the system, $\{\partial d/\partial EI^i\}$, $\{\partial \dot{d}/\partial EI^i\}$, $\{\partial \ddot{d}/\partial EI^i\}$ are the displacement, velocity and acceleration sensitivities with respect to the flexural rigidity of the ith element. Since the dynamic responses of the system have been obtained from Eq. (1), the sensitivities can be obtained numerically by direct integration from Eq. (4), then the response sensitivity matrix can be formed from these sensitivities.

3.2. Identification of system parameters

The identification problem can be expressed as follows to find the vector $\{EI\}$ of the system such that the calculated dynamic response R, for example, acceleration or displacement, etc., best matches the measured response $\hat{R}$, i.e.

$$\mathbf{Q}\{R\} = \{\hat{R}\}, \tag{5}$$

where the selection matrix $\mathbf{Q}$ is a constant matrix with elements of zeros or ones, matching the degrees-of-freedom corresponding to the measured response components.

Using the penalty function method [14], the identification equation can be expressed as

$$\{\Delta z\} = \mathbf{S}\{\Delta EI\}, \tag{6}$$

where $\{\Delta z\} = \{\hat{R}\} - \{R_{\mathrm{cal}}\}$ is the discrepancy in the measured and calculated response, $\{\Delta EI\}$ is the perturbation in the parameters, $\mathbf{S}$ is the time varying response sensitivity matrix, which contains the partial derivatives of the dynamic response with respect to the system parameters. For example, at time $t = t_i$, the sensitivity matrix can be expressed as

$$\mathbf{S}_{t=t_i} = \begin{bmatrix} \frac{\partial \ddot{d}_1(t_i)}{\partial EI^1} & \frac{\partial \ddot{d}_1(t_i)}{\partial EI^2} & \cdots & \frac{\partial \ddot{d}_1(t_i)}{\partial EI^i} & \cdots & \frac{\partial \ddot{d}_1(t_i)}{\partial EI^n} \\ \frac{\partial \ddot{d}_2(t_i)}{\partial EI^1} & \frac{\partial \ddot{d}_2(t_i)}{\partial EI^2} & \cdots & \frac{\partial \ddot{d}_2(t_i)}{\partial EI^i} & \cdots & \frac{\partial \ddot{d}_2(t_i)}{\partial EI^n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \frac{\partial \ddot{d}_i(t_i)}{\partial EI^1} & \frac{\partial \ddot{d}_i(t_i)}{\partial EI^2} & \cdots & \frac{\partial \ddot{d}_i(t_i)}{\partial EI^i} & \cdots & \frac{\partial \ddot{d}_i(t_i)}{\partial EI^n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \frac{\partial \ddot{d}_{NM}(t_i)}{\partial EI^1} & \frac{\partial \ddot{d}_{NM}(t_i)}{\partial EI^2} & \cdots & \frac{\partial \ddot{d}_{NM}(t_i)}{\partial EI^i} & \cdots & \frac{\partial \ddot{d}_{NM}(t_i)}{\partial EI^n} \end{bmatrix}, \tag{7}$$

where NM is the number of acceleration used in the identification.

Eq. (6) can be solved by simple least-squares method as follows:

$$\{\Delta EI\} = [\mathbf{S}^{\mathrm{T}}\mathbf{S}]^{-1}\mathbf{S}^{\mathrm{T}}\{\Delta z\}. \tag{8}$$

Like many other inverse problems, Eq. (8) is an ill-conditioned problem. In order to provide bounds to the solution, the damped least-squares method (DLS) [34] is used and singular-value decomposition is used in the pseudo-inverse calculation. Eq. (8) can be written in the following form in the DLS method:

$$\{\Delta EI\} = [\mathbf{S}^{\mathrm{T}}\mathbf{S} + \lambda\mathbf{I}]^{-1}\mathbf{S}^{\mathrm{T}}\{\Delta z\}, \tag{9}$$

where λ is the non-negative damping coefficient governing the participation of least-squares error in the solution. In the present study, L-curve method [35] is used to obtain the optimal regularization parameter.

Once the increment in the elemental flexural rigidity vector $\{\Delta EI\}$ has been calculated from Eq. (9), the updated vector can be obtained as follows:

$$\{EI\}_{j+1} = \{EI\}_j + \{\Delta EI\}_j, \tag{10}$$

where j denotes the jth iteration.

4. Numerical simulation

4.1. Free vibration analysis for the coupled system

First of all, free vibration analysis is performed for the coupled beam system. The system is assumed to be simply supported except otherwise specified, by setting the stiffness constants k_{sa} and k_{sb} to be a very large number, say 1.0×10^{10} and the stiffness constants k_{ra} and k_{rb} to be zeros in the numerical calculations. The physical parameters of the system are: Young's modulus $E = 34\,\text{GPa}$, mass density $\rho = 2800\,\text{kg/m}^3$, the width $b = 0.5\,\text{m}$, the depth $h = 1.0\,\text{m}$, sectional inertia moment $I = (1/12)bh^3$, the total length $L = 30\,\text{m}$. For the strongly coupled system, the coefficients of the two coupling springs are taken as: $k_s = 4.65 \times 10^9\,\text{N/m}$, $k_r = 4.65 \times 10^8\,\text{N m/rad}$; For the weakly coupled system, the coefficients of the two coupling springs are assumed to be: $k_s = 2.65 \times 10^{12}\,\text{N/m}$, $k_r = 2.65 \times 10^{11}\,\text{N m/rad}$. The coupled beam system is discretized into 20 Euler–Bernoulli beam elements with 21 nodes. The first eight natural frequencies of the intact strongly and weakly coupled beam are listed in Tables 1 and 2, respectively. From the Table 1 one can see, the natural frequencies cluster closely in each group, and in Table 2, the natural frequencies cluster equally in each group. The first eight mode shapes for the intact strongly and weakly coupled beam are shown in Figs. 2 and 3, respectively. From these figures one can

Table 1
The first eight natural frequencies (Hz) of the strongly coupled system.

Mode no.	Intact	Single damage	Multiple damages
1	10.24	10.23	10.15
2	10.96	10.95	10.85
3	33.43	33.30	33.01
4	35.49	35.36	34.97
5	70.21	69.84	69.23
6	73.95	73.62	72.81
7	120.83	120.31	119.44
8	126.27	125.82	124.74

Table 2
The first eight natural frequencies (Hz) of the weakly coupled system.

Mode no.	Intact	Single damage	Multiple damages
1	10.97	10.93	10.76
2	10.97	10.97	10.93
3	35.55	35.27	34.77
4	35.56	35.56	35.27
5	74.22	73.50	72.59
6	74.23	74.22	73.50
7	127.10	126.12	124.92
8	127.11	127.11	126.12

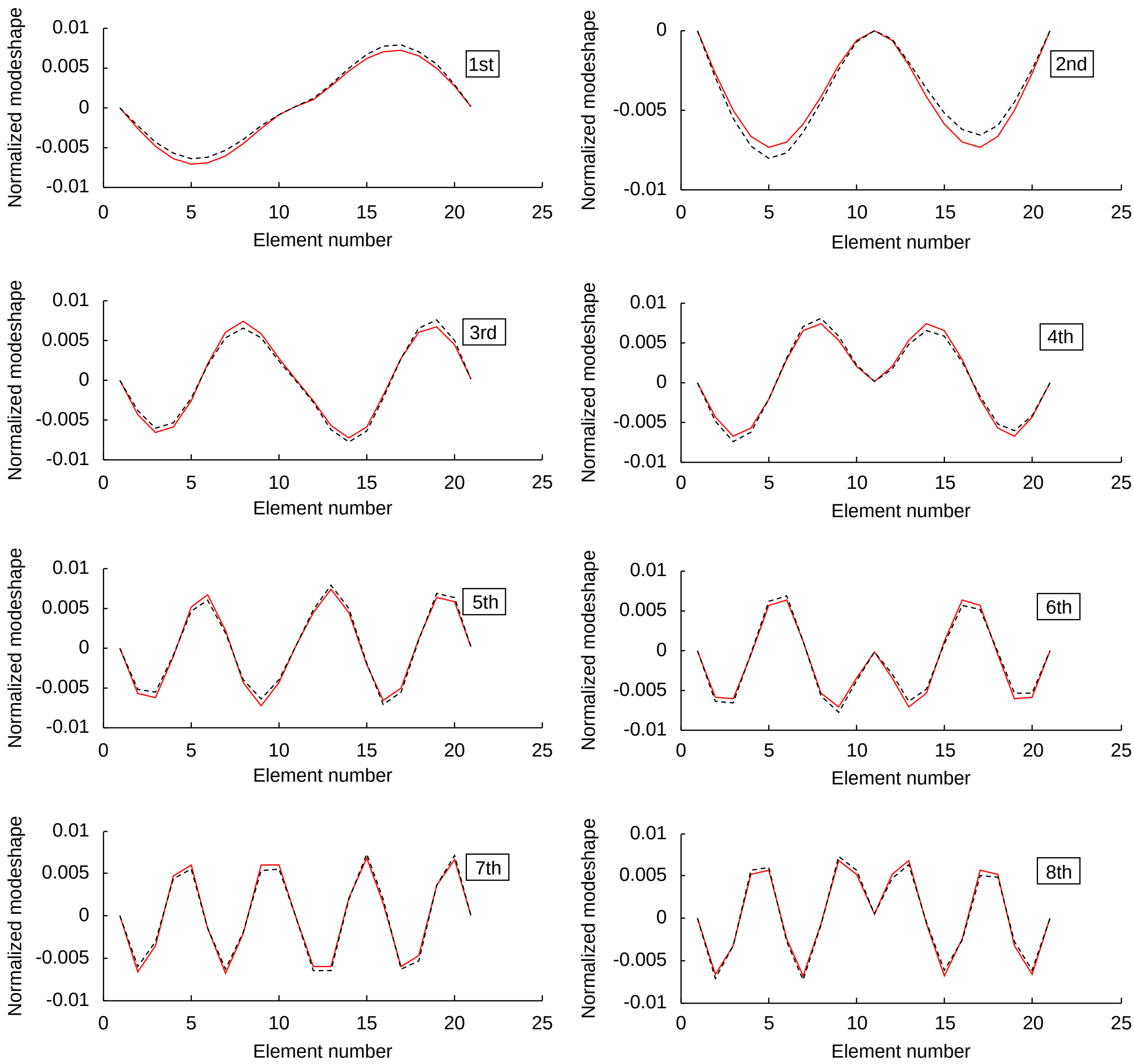

Fig. 2. The normalized mode shape of the strongly coupled intact and damaged beam system (—— intact and - - - damaged).

see, for the weakly coupled beam system, when there is a disorder (local damage), the mode localization phenomenon will occur. For the damaged system, the local damages are modeled by assuming a 10 percent reduction in the flexural rigidity in the 2nd, 11th and 19th elements of the system, respectively.

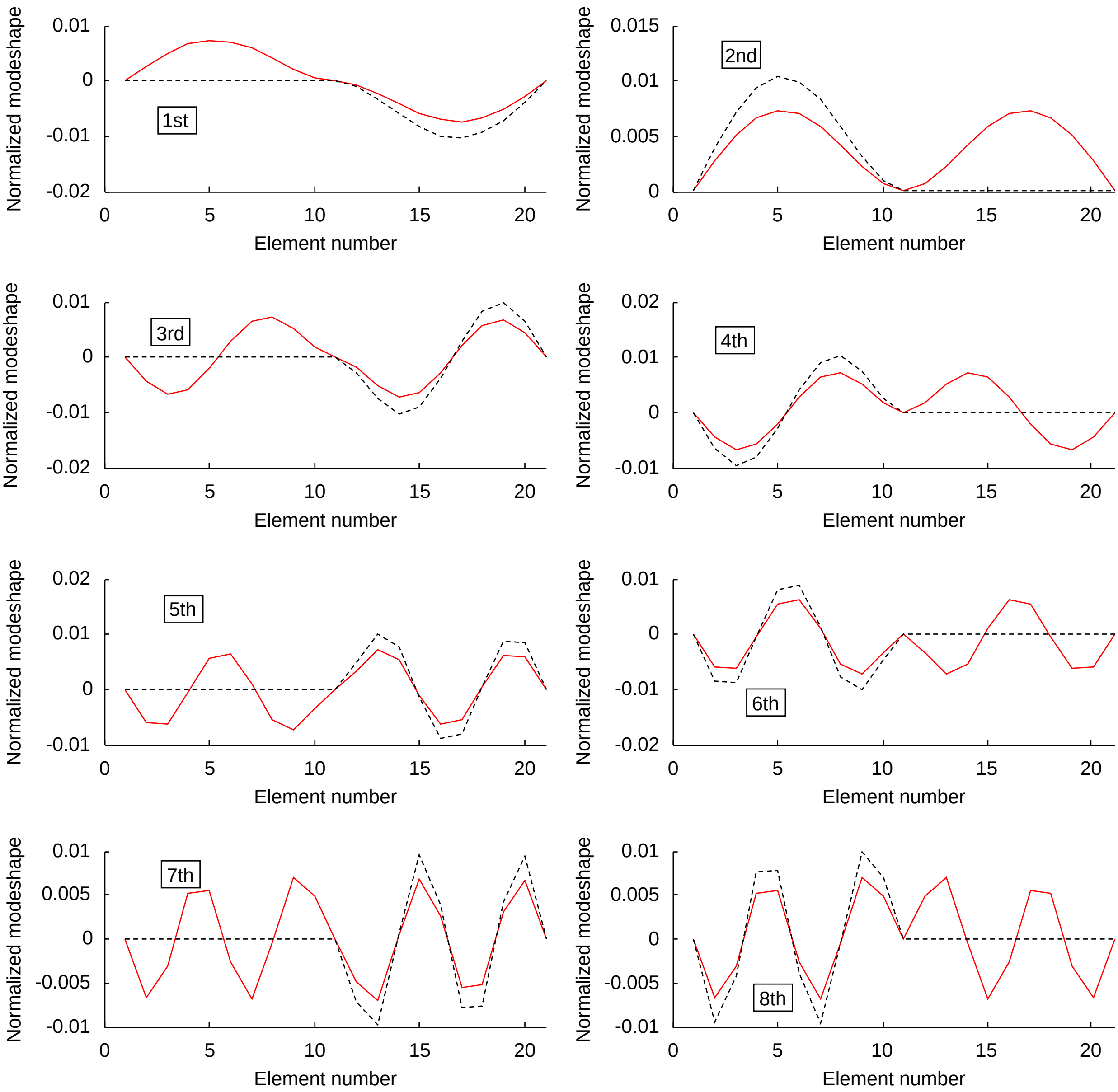

Fig. 3. The normalized mode shape of the weakly coupled intact and damaged beam system (—— intact and - - - damaged).

4.2. *Damage detection for the strongly coupled system*

In this section, the local damage(s) in the strongly coupled beam system with close natural frequencies is studied. The coefficients of the two coupling springs are assumed to be: $k_s = 4.65 \times 10^9$ N/m, $k_r = 4.65 \times 10^8$ N m/rad. The following three cases are investigated.

Study case 1: *single damage identification*: In this case, a local damage is simulated by a reduction in the flexural rigidity in the 2nd element of the system by 10 percent. A moving force is assumed to be $F(t) = 16\,000(1+0.1\sin(5\pi t) + 0.05\sin(15\pi t))N$, which moves from the left support of the coupled beam system to the right support with a constant speed 5 m/s. Newmark integration method is adopted to obtain the dynamic response of the system and the first two modal damping ratios are assumed to be 0.01. The time step is taken as 0.005 s and the time duration is 6 s, so there are totally 1200 time response data in the identification, which is much greater than the number of unknowns 20. Only three accelerometers locate at the 6th, 10th and 17th nodes of the system is used to collect the acceleration responses for damage detection. In this case, the measurement data are assumed to be noise free. The local damage was identified with good accuracy as shown in Fig. 4 in 11 iterations with the optimal regularization parameter λ equal to 1.22×10^{-9}.

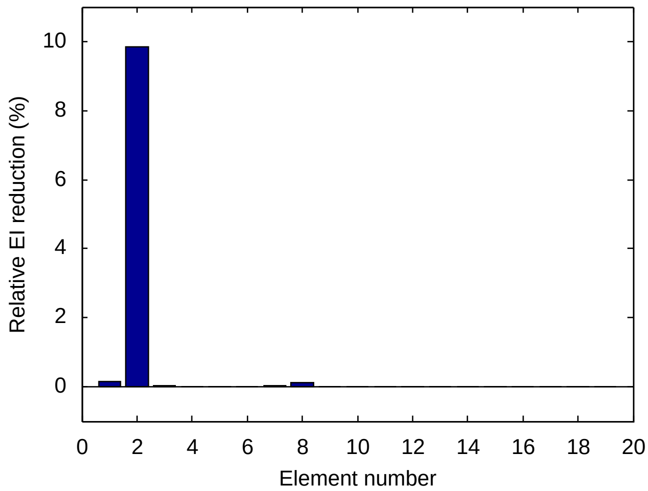

Fig. 4. Single damage identification for strongly coupled system (noise free).

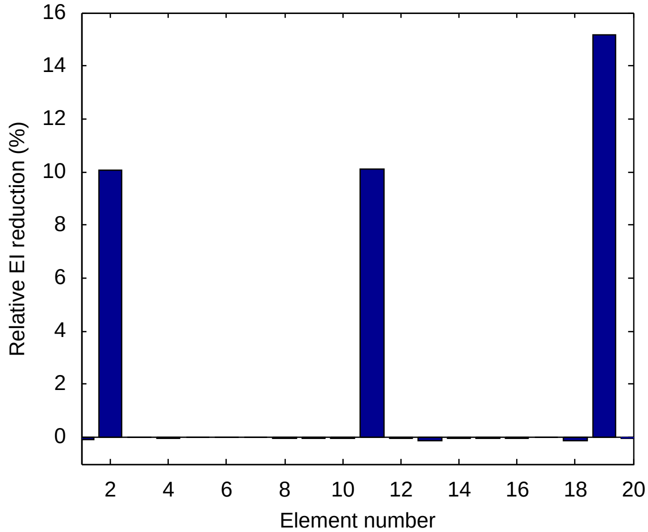

Fig. 5. Multiple damages identification in strongly coupled system (noise free).

Study case 2: *multiple damages identification with noise free measurement data*: In this case, multiple local damage detection is studied. Three local damages are introduced into the system, which locate at the 2nd, 11th, and 19th elements with a reduction in the flexural rigidities by 10, 10 and 15 percent, respectively. The same moving force and the same three accelerometers as Study case 1 are used in the damage detection. These three local damages are identified successfully in 14 iterations with the optimal regularization parameter λ equal to 1.67×10^{-9}. Fig. 5 shows the identified results.

Study case 3: *multiple damages identification with noisy measurement data*: In this case the effect of artificial measurement noise on the identified results is investigated. A normally distributed random error with zero mean and a unit standard deviation is added to the calculated acceleration to simulated the "noisy" measurement as shown below:

$$\hat{\ddot{d}} = \ddot{d}_{\mathrm{cal}} + E_p * N_{\mathrm{oise}} * \mathrm{var}(\ddot{d}_{\mathrm{cal}}), \tag{11}$$

where $\hat{\ddot{d}}$ is the vector of polluted acceleration response; E_p is the noise level ; N_{oise} is a standard normal distribution vector with zero mean and unit standard deviation; $\mathrm{var}(\cdot)$ is the variance of the time history. In the simulation, 5 and 10 percent noise level is added to the calculated responses to simulate the measured noisy data, respectively. The same moving force and the same three accelerometers as Study case 1 are used in the damage detection. Three simulated local damages are identified successfully in 17 and 19 iterations with the optimal regularization parameter λ equal to 8.9×10^{-9} and

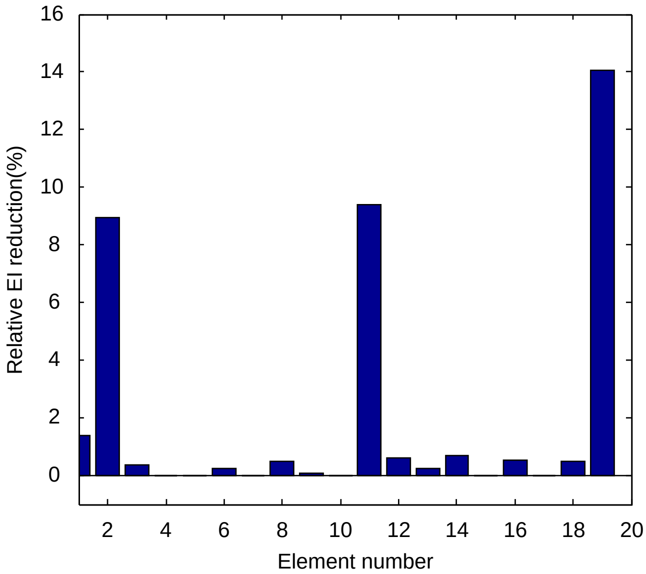

Fig. 6. Multiple damages identification in strongly coupled system with noisy measurements (5 percent noise level).

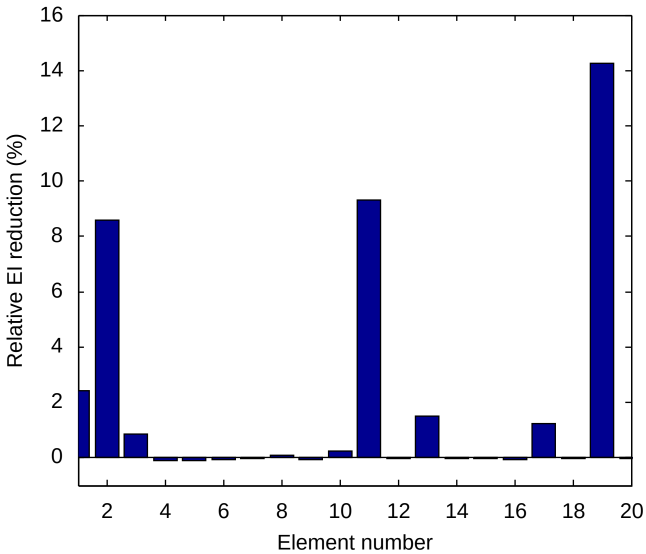

Fig. 7. Multiple damages identification in strongly coupled system with noisy measurements (10 percent noise level).

9.2×10^{-9} for 5 and 10 percent noise level, respectively. The identified results with different noise levels are shown in Figs. 6 and 7. For 5 percent noise level, the maximum error in the identification is 1.2 percent in the 1st element and for 10 percent noise level, the maximum error is 2.3 percent in the 1st element. From these figures one can see, the proposed method is not sensitive to the artificial measurement noise.

4.3. Damage detection in the weakly coupled beam system

Now we move to identify the local damage(s) in the weakly coupled beam system with repeated natural frequencies. As we know, mode localization phenomena would occur in the weakly coupled beam system when there is local damage(s) in the system. In this section, the effect of the mode localization on the damage detection is examined. The coefficients of the two coupling springs are changed as: $k_s = 2.65 \times 10^{12}\,\mathrm{N/m}$, $k_r = 2.65 \times 10^{11}\,\mathrm{N\,m/rad}$. The following three cases are studied.

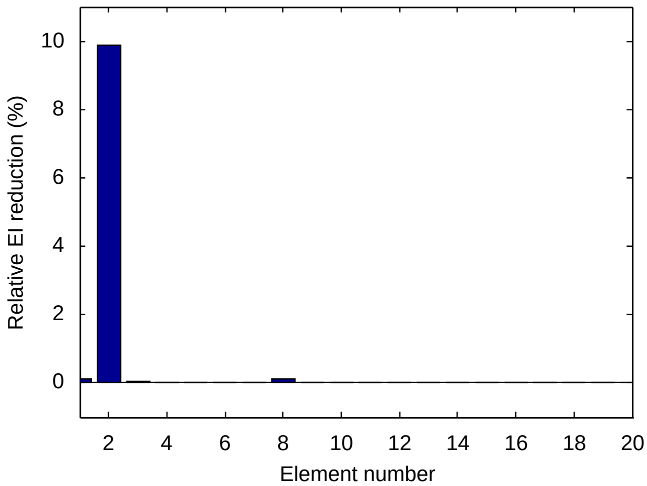

Fig. 8. Single damage identification weakly coupled system (noise free).

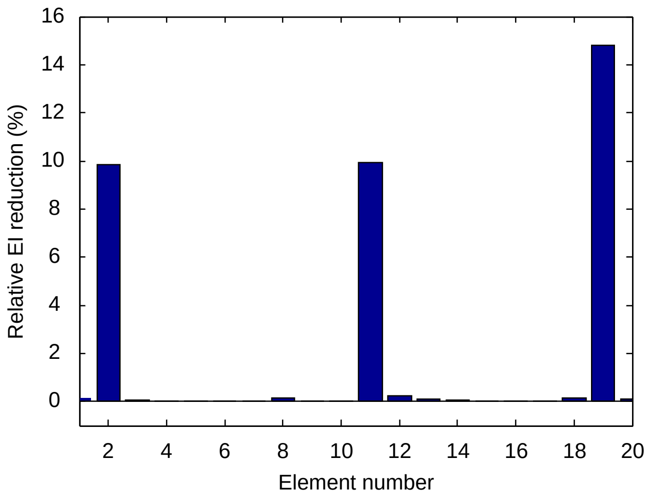

Fig. 9. Multiple damages identification in weakly coupled system (noise free).

Study case 4: *single damage identification*: Study case 1 is re-examined here. The same moving force and measurements as Study case 1 are used for damage detection. Again, this local damage has been identified successfully as shown in Fig. 8 in 12 iterations with the optimal regularization parameter equal to 1.81×10^{-9}. This shows the mode localization has little effect on the damage detection.

Study case 5: *multiple damages identification with noise free measurement data*: Study case 2 is re-examined. The same moving force and measurements as Study case 2 are used for damage detection. Fig. 9 shows the identified results for the system after 16 iterations with optimal regularization parameter λ equal to 2.3×10^{-9}. From this figure one can see, the three locations of the local damages in the system have been identified successfully without any false alarm in the neighborhood of those damaged elements. This further shows the effectiveness of the proposed method for damage detection.

Study case 6: *Multiple damages identification with noisy measurement data*: In this study case, Study case 5 is re-examined. The same moving force and the same measurements as the last study case are used in the identification. Again, 5 and 10 percent noise level is added to the calculated responses to simulate the measured noisy data, respectively. Those three local damages are identified with very good accuracy in 19 and 23 iterations with the optimal regularization parameter λ equal to 6.9×10^{-8} and 7.4×10^{-8} for noise level 5 and 10 percent, respectively. Figs. 10 and 11 show the identified results with different noise levels. For 5 percent noise level, the maximum error in the identification is 1.8 percent in the 13th element and for 10 percent noise level, the maximum error is 2.5 percent in the 13th element.

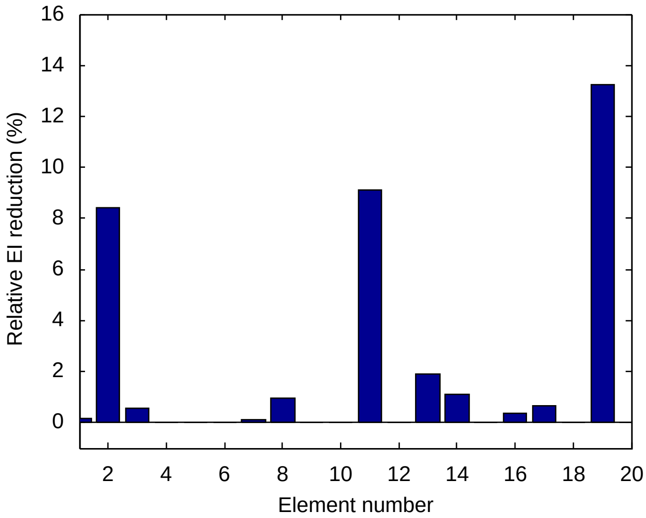

Fig. 10. Multiple damages identification in weakly coupled system with noisy measurements (5 percent noise level).

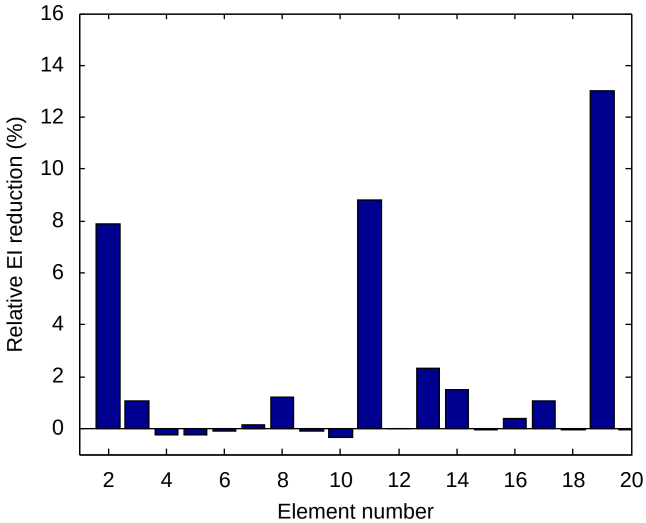

Fig. 11. Multiple damages identification in weakly coupled system with noisy measurements (10 percent noise level).

4.4. Damage detection for the coupled beam system with elastic supports

The coupled beam system with more general boundary conditions is examined in this case; the coupled beam system is arbitrarily coupled via a set of linear and rotational springs. The coefficients of those coupling springs are arbitrarily taken as: $k_{sa} = k_{sb} = 2 \times 10^5\,\mathrm{N/m}$, $k_{ra} = k_{rb} = 10^4\,\mathrm{N\,m/rad}$, $k_s = 8.5 \times 10^9\,\mathrm{N/m}$, $k_r = 8.5 \times 10^8\,\mathrm{N\,m/rad}$. In the identification, the same moving force and the same measurements as the last study case are used. Again, 5 and 10 percent noise level is added to the calculated responses to investigate the effect of measurement noise on the identified results. Those three local damages are identified successfully in 19 and 24 iterations with the optimal regularization parameter λ equal to 4.9×10^{-8} and 5.3×10^{-8} for noise level 5 and 10 percent, respectively. The identified results with different noise levels are shown in Figs. 12 and 13. For 5 percent noise level, the maximum error in the identification is 1.5 percent in the 1st element and for 10 percent noise level, the maximum error is 3.7 percent in the 1st element. This shows the robustness of the proposed method for damage detection.

A further case is studied to check the effect of heavy measurement noise on the results of damage detection. A heavy noise with 30 percent noise level is added to the calculated responses to simulate the measured noisy response. Again, three local damages are identified successfully in 27 iterations with the optimal regularization parameter λ equal to 5.4×10^{-8}. The identified result is shown in Fig. 14. Even under such noise level, the maximum error in the identification

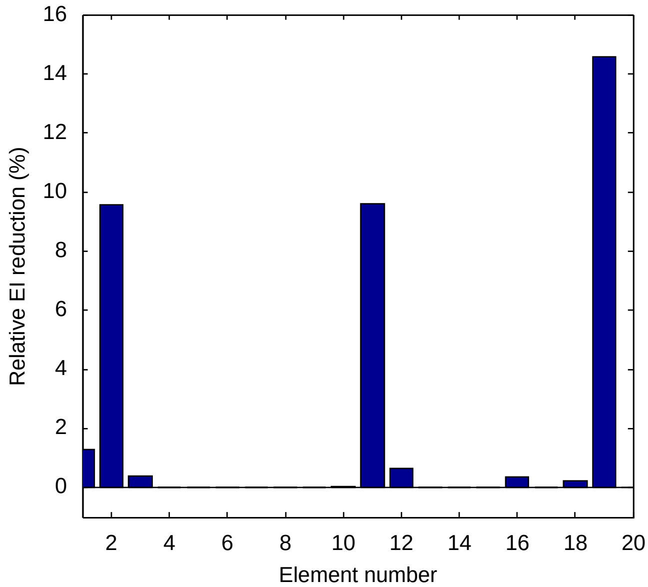

Fig. 12. Multiple damages identification in generally coupled system with noisy measurements (5 percent noise level).

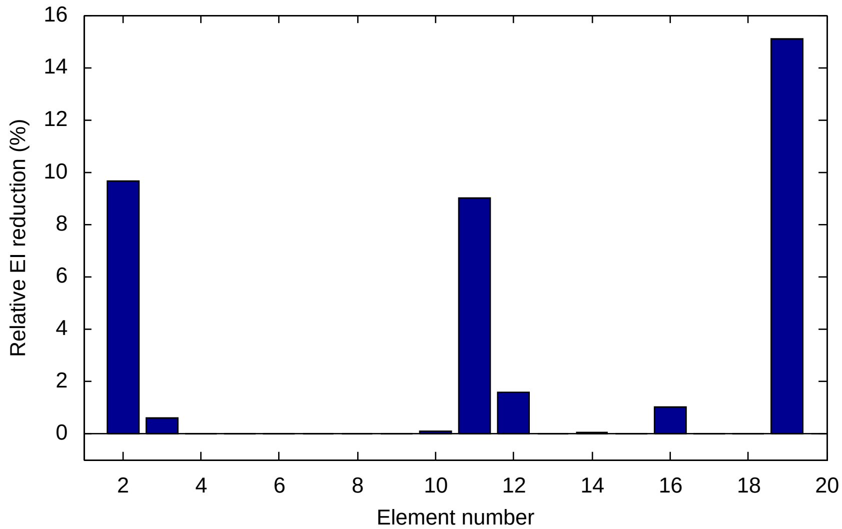

Fig. 13. Multiple damages identification in generally coupled system with noisy measurements (10 percent noise level).

is around 4 percent in the 12th element. This shows that the proposed method is insensitive to artificial measurement noise.

5. Conclusions

The local damages identification problem for the strongly and weakly coupled beam systems with close and repeated natural frequencies is studied in the present study. It is found that the mode localization phenomenon would occur due to local damage in a dual-span weakly coupled beam system. However, the mode localization has little effect on the damage detection. Numerical simulation shows that the proposed method is effective for local damage identification in both strongly and weakly coupled systems. And it is found the present method is not sensitive to artificial measurement noise, this shows the proposed method has the potential for practical damage detection.

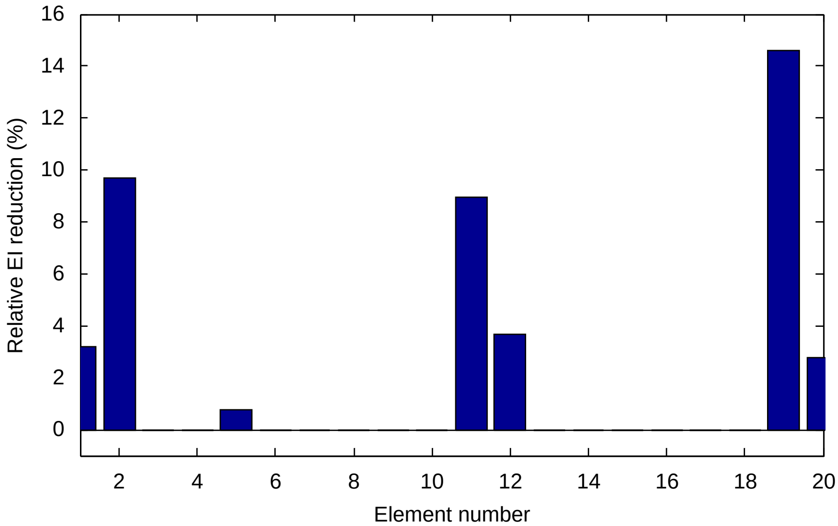

Fig. 14. Multiple damages identification in generally coupled system with noisy measurements (30 percent noise level).

Acknowledgments

This work is supported in part by the NSFC (10772202), Guangdong Province Science and Technology Program (2007B030402002) and a Foundation from Ministry of Education of China (200805581018).

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